

Cálculo Integral

Substituição e Partes

1. Resolva os integrais, efectuando a substituição indicada

a) $\int \frac{x}{\sqrt{x+1}} dx \quad t = \sqrt{x+1}$

b) $\int \frac{\cos^3(x)}{1 - \sin(x)} dx \quad t = \sin(x)$

c) $\int \frac{5^x(1+5^x)}{\sqrt{1-5^{2x}}} dx \quad t = 5^x$

d) $\int \frac{2-3\ln(x)}{x(5+\ln^2(x))} dx \quad t = \ln(x)$

e) $\int \frac{1}{\sqrt{(1-x^2)^3}} dx \quad x = \sin(t)$

f) $\int \frac{1}{x^2\sqrt{x^2-9}} dx \quad x = 3\sec(t)$

g) $\int \frac{1}{\sqrt{9+(x-2)^2}} dx \quad x-2 = 3\tan(t)$

2. Calcule os integrais

a) $\int x e^{-3x} dx$

b) $\int x \sin(2x) dx$

c) $\int \arccos(3x) dx$

d) $\int \ln(1+x^2) dx$

e) $\int \sqrt[3]{x+5} \ln(x+5) dx$

f) $\int x \operatorname{arctg}\left(\frac{x}{2}\right) dx$

g) $\int (x^2+1) e^{-3x} dx$

h) $\int x \cos^2(2x) \sin(2x) dx$

i) $\int x \tan^2(2x) dx$

j) $\int \frac{1}{x^3} e^{\frac{1}{x}} dx$

k) $\int x^3 \sec^2(x^2+3) dx$

l) $\int x \ln^2(2x) dx$

3. Determine a expressão analítica da função f , sabendo que o seu gráfico passa no ponto $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$ e que $f'(x) = \sec^2(x) \ln(\sec(x))$.

4. Calcule

a) $\int \frac{1}{\sqrt{x}} \arccos(\sqrt{x}) dx \quad t = \sqrt{x}$

b) $\int \frac{1}{x^3} \cot^2\left(\frac{1}{x}\right) dx \quad t = \frac{1}{x}$

Soluções

1. a) $\frac{2}{3}\sqrt{(x+1)^3} - 2\sqrt{x+1} + C$
 b) $\operatorname{sen}(x) + \frac{1}{2}\operatorname{sen}^2(x) + C$
 c) $\frac{1}{\ln(5)} \left[\operatorname{arcsen}(5^x) - \sqrt{1-5^{2x}} \right] + C$
 d) $\frac{2\sqrt{5}}{5} \operatorname{arctg} \left(\frac{\ln(x)}{\sqrt{5}} \right) - \frac{3}{2} \ln(5 + \ln^2(x)) + C$
- e) $\frac{x}{\sqrt{1-x^2}} + C$
 f) $\frac{\sqrt{x^2-9}}{9x} + C$
 g) $\ln \left| \frac{x-2 + \sqrt{9+(x-2)^2}}{3} \right| + C$
2. a) $-\frac{1}{3}x e^{-3x} - \frac{1}{9}e^{-3x} + C$
 b) $-\frac{1}{2}x \cos(2x) + \frac{1}{4} \operatorname{sen}(2x) + C$
 c) $x \arccos(3x) - \frac{1}{3}\sqrt{1-9x^2} + C$
 d) $x \ln(1+x^2) - 2x + 2\operatorname{arctg}(x) + C$
 e) $\frac{3}{4}\sqrt[3]{(x+5)^4} \ln(x+5) - \frac{9}{16}\sqrt[3]{(x+5)^4} + C$
 f) $\frac{x^2}{2} \operatorname{arctg} \left(\frac{x}{2} \right) + 2 \operatorname{arctg} \left(\frac{x}{2} \right) + C$
- g) $-\frac{1}{3}x^2 e^{-3x} - \frac{2}{9}x e^{-3x} - \frac{11}{27}e^{-3x} + C$
 h) $-\frac{1}{6}x \cos^3(2x) + \frac{1}{12} \operatorname{sen}(2x) - \frac{1}{36} \operatorname{sen}^3(2x) + C$
 i) $\frac{x}{2} \operatorname{tg}(2x) + \frac{1}{4} \ln |\cos(2x)| - \frac{1}{2}x^2 + C$
 j) $e^{\frac{1}{x}} - \frac{1}{x} e^{\frac{1}{x}} + C$
- k) $\frac{1}{2}x^2 \operatorname{tg}(x^2+3) + \frac{1}{2} \ln |\cos(x^2+3)| + C$
 l) $\frac{1}{2}x^2 \ln^2(2x) - \frac{1}{2}x^2 \ln(2x) + \frac{1}{4}x^2 + C$
3. $f(x) = \operatorname{tg}(x) \ln(\sec(x)) - \operatorname{tg}(x) + x + \sqrt{3} - \sqrt{3} \ln(2)$
4. a) $2\sqrt{x} \arccos(\sqrt{x}) - 2\sqrt{1-x} + C$
 b) $\frac{1}{x} \operatorname{cotg} \left(\frac{1}{x} \right) - \ln \left| \operatorname{sen} \left(\frac{1}{x} \right) \right| + \frac{1}{2x^2} + C$